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Nonlinear Finite Element Analysis of Reinforced Concrete Brackets

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Abstract- The research investigates the behavior and ultimate load of reinforced concrete brackets for normal strength concrete. A theoretical study using finite element method via computer program ANSYS 16.1 is used to create three-dimension models for twenty specimens based on experimental study. These twenty specimens of normal strength of concrete divided into seven series, the variables investigated are shear span to effective depth with and without stirrups, amount of the shear reinforcement (stirrups), the ratio of outside depth to the total depth of the brackets (k/h), the compressive strength of the concrete and the main reinforcement ratio.

The results show that the agreement between the finite element method and experiments is good for results of ultimate load, with maximum difference ratio was 7%.It is observed that increasing the compressive strength of concrete, amount of main and secondary (stirrups) reinforcement and the ratio of outside to the total depth (k/h) causes the increase of the ultimate load. It is also found that decreasing the shear span ratio to the effective depth (a/d) causes increasing the ultimate load.

Results arrived at in this study also show that the crack patterns started as flexural cracks and with increasing the applied load diagonal cracks appeared to lead the brackets to fail by Shear Beam Failure for the specimens with stirrups reinforcement and Diagonal Tension Failure for the specimens without stirrups.

The comparison between the results of finite element and ACI 318-77 is conducted, and reasonable agreement for the factors of safety 1.02 to 1.7 and were observed.

1 Introduction

Brackets or corbels are short-haunched cantilevers emanating from the inner face of columns or concrete walls to sustain heavy concentrated loads of precast beams, gantry girders, and other precast system loads. The ratio of shear span to depth is often less than 1.0. The behavior of nonlinear stress of the short member is therefore influenced by the shear deformation in the elastic range and accordingly the shear strength of the section becomes a parameter of importance for design consideration [1]. Since a corbel forms the main part of the connection between a beam and a column, it should be strengthened more than either the beam or the column

Mattock et al. [1] in 1976, presented an empirical study on the behavior of 28 reinforced concrete brackets. 26 of these brackets contained horizontal stirrups. The ratio of vertical to horizontal load (V_u/H_u), shear span to effective depth ratio (a/d), the kind of aggregate and the amount of main tensile and stirrup reinforcement were as a variables. Regarding brackets without stirrups, essentially only one diagonal tension crack was formed and very sudden failure occurred. As a result, a diagonal splitting mode of failure happened. Two of 26 brackets with horizontal stirrups had flexural failure, because it had wide opening of the flexural cracks, while the diagonal tension cracks remained fine. The failure of the remaining brackets was categorized as "beam-shear" kind failure. Here, the flexural cracks remained fine and failure had the feature of widening of one or more diagonal tension cracks and the shear-compression failure of the concrete near the intersection of the sloping corbel face and the column face. Failure was quite sudden, but less brittle and with more precision than in those of diagonal tension failure of brackets without horizontal stirrups.

Foster et al. [3] in 1996, conducted a study on 30 high strength concrete corbels.

They examined the effect of the shear span to depth ratio, compressive strength and the secondary reinforcement. They concluded that:-

1- High strength concrete bracket behavior is like the behavior of normal strength concrete corbel.
2- Availability of secondary reinforcement reduces crack width, improves ductility, and changes the failure mode from diagonal splitting to compression strut failure.
3- Main steel amount and availability of secondary reinforcement do not appear to influence the load at first crack.

Renuka et al [4] in 1993, used nonlinear finite element to analyze reinforced concrete brackets. An elastic-plastic-cracking constitutive formulation using Huber-Hencky-Mises yield surface augmented with a tension cut-off-was employed. Smeared-fixed cracking with mesh-dependent strain softening model was employed to obtain objective results. Multiple non-orthogonal cracking and opening and closing of cracks were permitted. The results obtained from the nonlinear finite element analysis were compared with experimental results.

2- ACI 318-77 Provision [5]

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The design of concrete bracket is mainly for vertical load that is transferred by girders or beams. The ACI code provision for brackets are based on the results obtained from more than 200 tests, and the formulas prescribed are simplified approximations of experiments given by Kriz and Raths [6]

The ACI code gives requirements for the brackets that are subjected to vertical load only and having shear span to effective depth (a/d) equal to unity. The computed ultimate load (Vu) may not exceed the value from ACI 318-77 formula, which is Eq. (1)

$$Vu = 0.54(1 - 0.5a/d)(1 + 64\rho_v)\sqrt{f'c}\ bd \qquad \text{....1}$$

Where

$$\rho_v = \frac{As + Ah}{bd} \qquad \text{....2}$$

Ah = area of shear reinforcement (mm^2)

d= effective depth of the bracket (mm)

b= width of bracket (mm)

3- Modeling the Reinforced Concrete Brackets

The first step for modeling the specimens in finite element analysis via ANSYS is building the model in which the bracket geometry is made then classified into finite elements which are joined together by nodes. To represent the specimens, it is necessary to give definitions of the element types, element real constants, materials properties and the model geometry. Doing this step, one can define the analysis type and options, apply boundary conditions and loads, select load step options then initiate the finite element solution leading to the calculation of the stresses and strains at integration points of elements.

3.1 The Model Geometry

The concrete, reinforcement, plates and the supports were modeled by creating the nodes on the working plane of ANSYS 16.1, then building the elements through the nodes. The following element types available in the ANSYS element library are used for modelling the brackets:
Solid 65 element has been used to create the concrete through the nodes.
Link180 element has been used to create the reinforcement (main and stirrups), also through the nodes which are connected the concrete elements.
Solid185 element has been used to represent the supports and load plates

3.2 Meshing

It is recommended to use a rectangular mesh for the SOLID 65 element. The support and the steel plate elements were set to be consistent with concrete element, and the reinforcement elements were created through the nodes of the concrete elements mesh. Triangular elements are used at sloping edges of the models as shown in Fig (1)

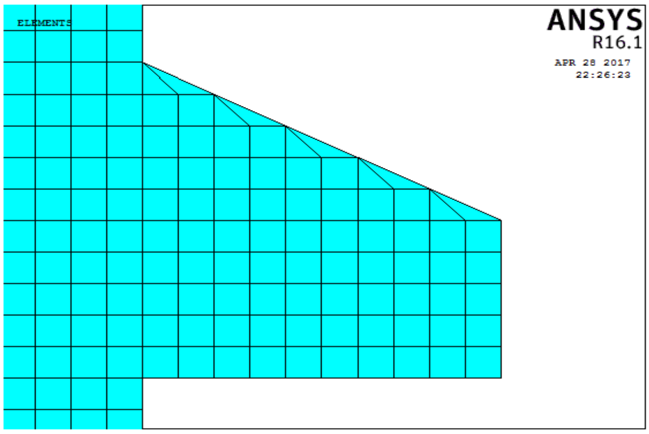


Fig 1. Edge of the Bracket

3.3 Load Application

All specimens in the four case studies were analyzed an inverted position as shown in Fig (2) where the load is applied at the top of the column. It is noted that the applied load is double the load applied on each bracket.

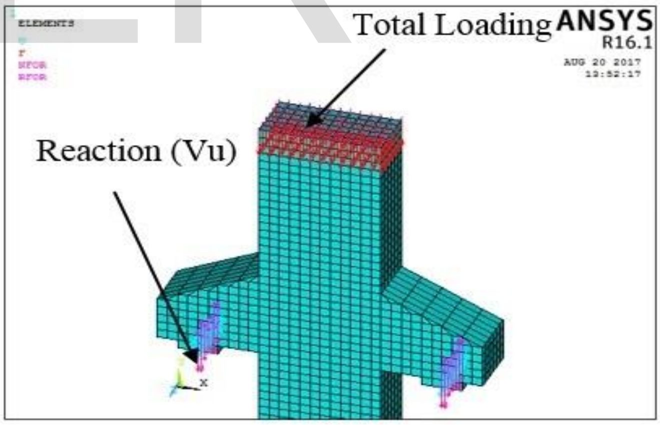


Fig 2. Loading

3.4 Boundary Conditions

In order to avoid a non-singular solution, it is necessary to constrain the model by applying the correct displacement boundary conditions.

The nodes along a single line on the one of the steel plates were constrained in the y-direction thus simulating a roller support, whereas the nodes along a line on the other plate were constrained in the y- and z- directions simulating a hinged support. Fig (3) shows the boundary condition of the bracket models.

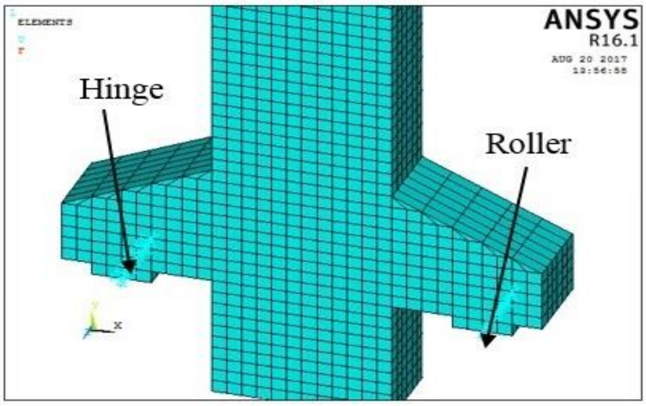
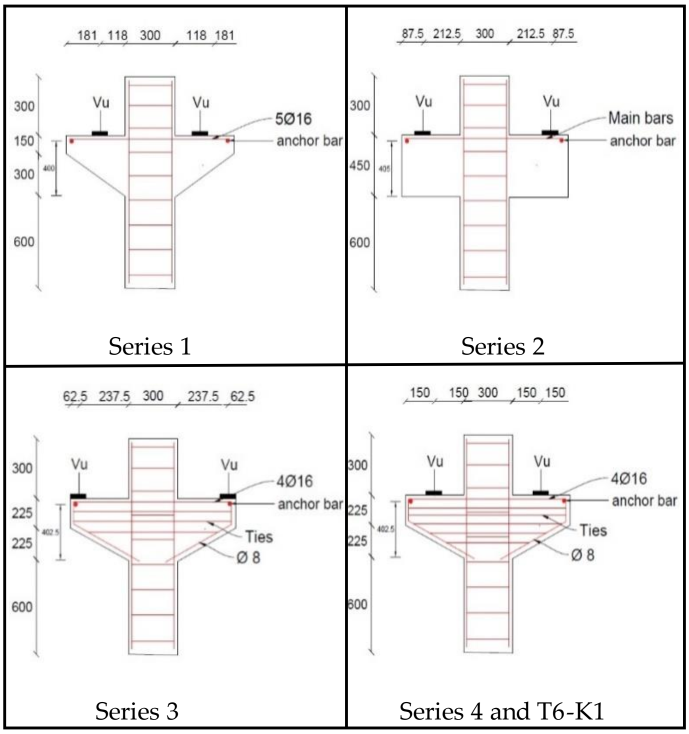


Fig 3. Boundary Conditions

4 Case study

In this part of the study, the investigation done is based on the experimental study that is presented by Rath and Kriz[6] ; their experimental study included 71 specimens subjected to combined vertical and horizontal loads and 114 specimens subjected to vertical load only and 15 specimens to investigate, the effect of additional column loads.
In this study, twenty specimens of 114 specimens which are subjected to vertical load are considered.

The variables considered in this study were shear span to effective depth with and without stirrups, amount of the shear reinforcement (stirrups) the ratio of outside depth to the total depth of the brackets (k/h), the compression strength of the concrete and the main reinforcement ratio. All details of the twenty specimens are given in Table (1) and Fig (4)

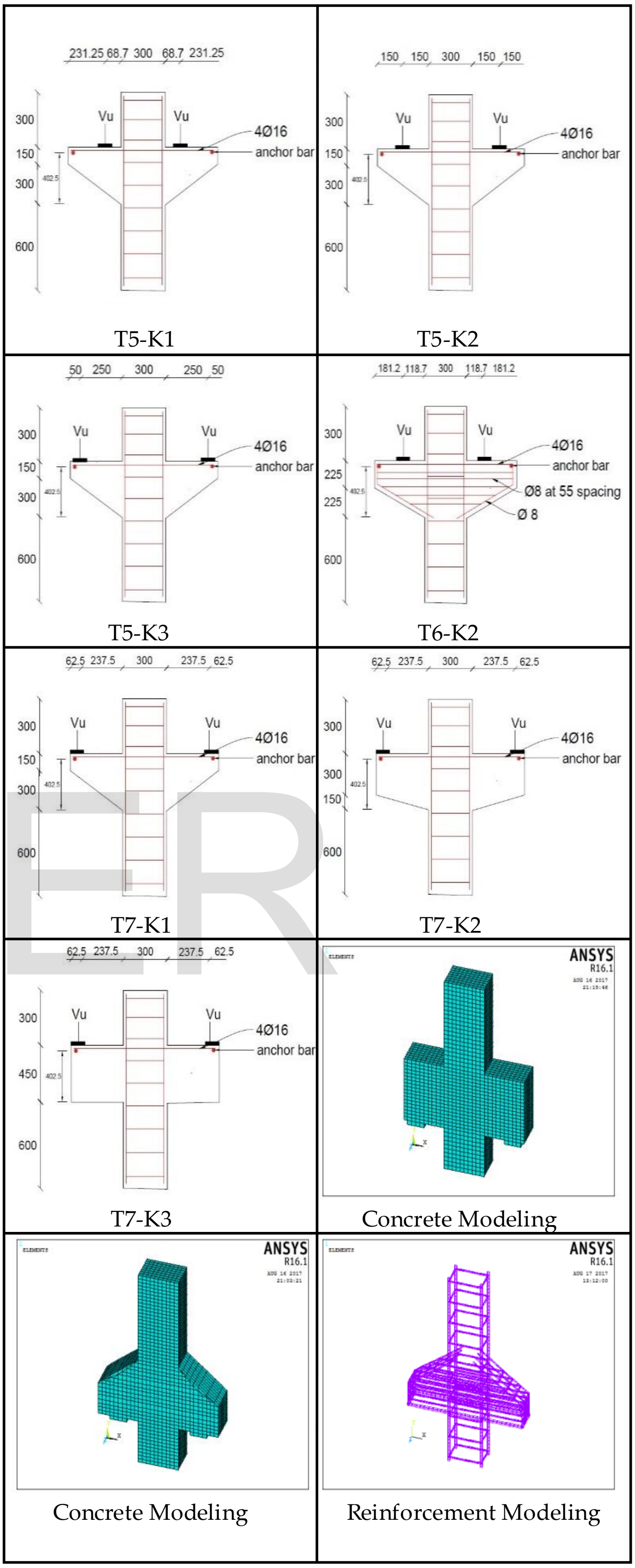


Series 1

Series 2

Series 3

Series 4 and T6-K1



T5-K1

T5-K2

T5-K3

T6-K2

T7-K1

T7-K2

T7-K3

Concrete Modeling

Concrete Modeling

Reinforcement Modeling

Fig 4. Structural and Geometry Details of the Specimens

Notes

1- All dimensions in mm.

2- *All specimens have the same details of column (300 x 200)mm.*
3- *The details of reinforcement of the column (4-D16 for the main reinforcement and D8 @ 150 mm spacing for the stirrups)*
4- f_y *= 414 MPa has been used for column reinforcement (main and stirrups)*
5- *Steel load plates (125 x 200x 20)mm for series 1, 2, 3,6, 4 and 7.*
And (75 x 200 x20) mm for series 5
6- *The stirrups were uniformly in the two-thirds of the effective depth*[6].
7- *The width of the brackets (b) is constant and equals 200 mm .*
8- *The diameter of anchor bar is equal to the diameter of main reinforcement*

5- Presentation and Discussion of the Results

As explained, the applied load is double of the ultimate load (Vu), because there are two brackets with column and the load was applied on the top of the column, so the value of the failure load which is calculated by ANSYS has been divided by 2 for the purpose of obtaining the value of ultimate load (Vu) of the brackets to compare with experimental values and theoretical values according ACI. The results considered are ultimate load or the load at failure, deflection, stress of reinforcement, and cracks pattern.

5.1 Ultimate and Cracking Loads

The finite element results obtained for ultimate, 1st cracking loads and the deflection at failure for the case study considered are given in Table (2). The results show good agreement with the experimental values and those calculated according to the ACI, with maximum difference 7%.

The difference between the experimental results and the results obtained from the analysis of the specimens of brackets by ANSYS program may be attributed to the assumption of perfect bond between reinforcement and concrete while in the actual specimens this does not exist. In addition to the fact that toughening mechanisms at the crack faces which may also slightly extend the failures of the experimental beams before complete collapse will happen. The finite element models do not include such mechanisms.

ACI 318-77[4] equation considered all the variables except the ratio of k/h. The ultimate load values that obtained from ACI equation indicated that the ultimate load increases when increasing compressive strength and amount of main and secondary reinforcement, also the ultimate load increases with decreases the a/d ratio.

According to comparison between the results of finite element and ACI code equation, it is observed that ACI code equation provided good factor of safety which was 1.02 – 1.7.

5.1.1 Effect of Compressive Strength of the Concrete on the Ultimate Load of the Brackets

The results of series one of the case study that investigates the effect of the compressive strength for normal strength concrete, shows that the increase of the compressive strength from 14 MPa to 44 MPa leads to increase the ultimate strength about 51%. Fig (5) shows the variation of ultimate load with *f'c* and verification of the experimental study

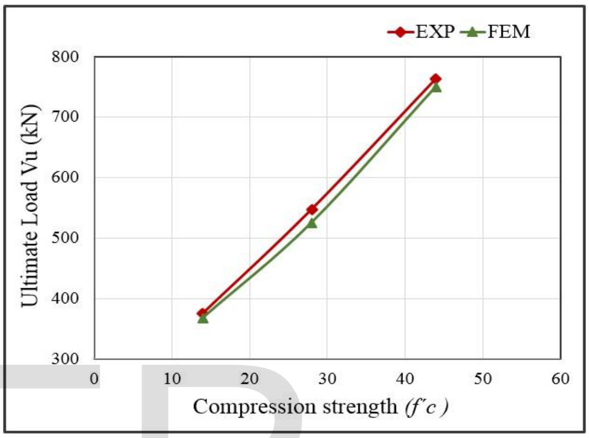


Fig (5) Variation the Ultimate Load with Compressive Strength of Series One

5.1.2 Effect of Main Reinforcement on the Ultimate Load of the Brackets

Investigation of the effect of main reinforcement ratio for brackets with normal strength concrete. Reveals that the ultimate strength increases by 43.15% for a corresponding increase from 0.48% - 1.23% in the main reinforcement ratio, which is in good agreement with experimental results, Fig. (6).

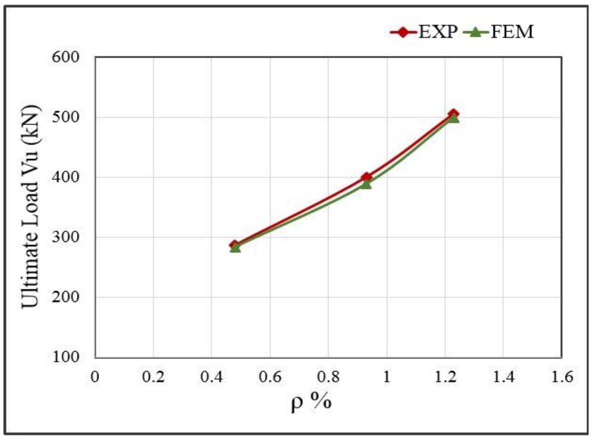
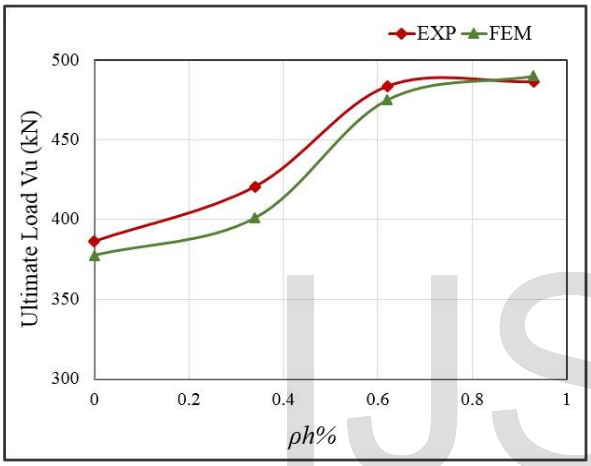


Fig (6) Variation the Ultimate Load with Main Reinforcement Ratio

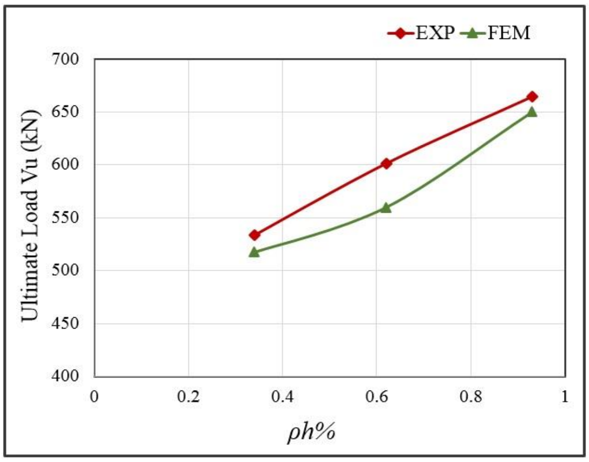
5.1.3 Effect the Horizontal Shear Reinforcement (Stirrups) on the Ultimate Load of the Brackets

It appears from the results of the series three of the case study whose specimens have a/d equal to 0.590 and as shown in Fig (7 a), good agreement exists between the results of finite element and the experimental results, it is found that the ultimate strength of the brackets increased about 20.6% when the ratio of stirrups reinforcement changed from no stirrup reinforcement to 0.93 %.

The fourth series and specimen T6-K1 have been analyzed to investigate the effect of the stirrup reinforcement ratio but with different a/d ratio equal to 0.372, and it is found from Fig (7b), the ultimate strength of the brackets increased about 20.33% when the ratio of stirrups reinforcement increased from 0.34% to 0.93%.



a- Specimens with *a*/*d*=0.590



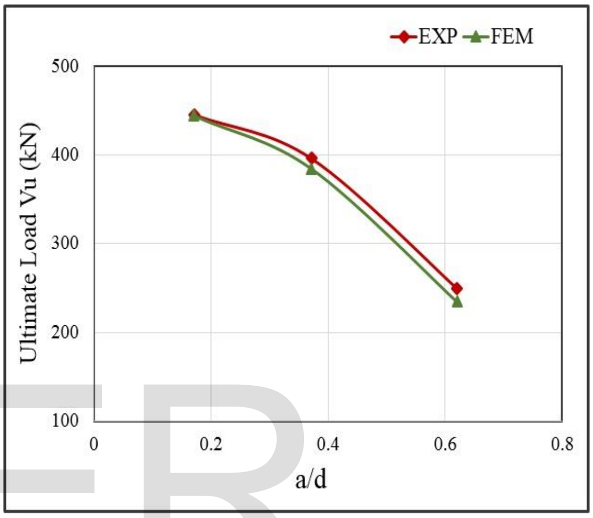
b- Specimens with a/d= 0.372

Fig (7) Variation the Ultimate Load with the Ratio of Stirrups Reinforcement

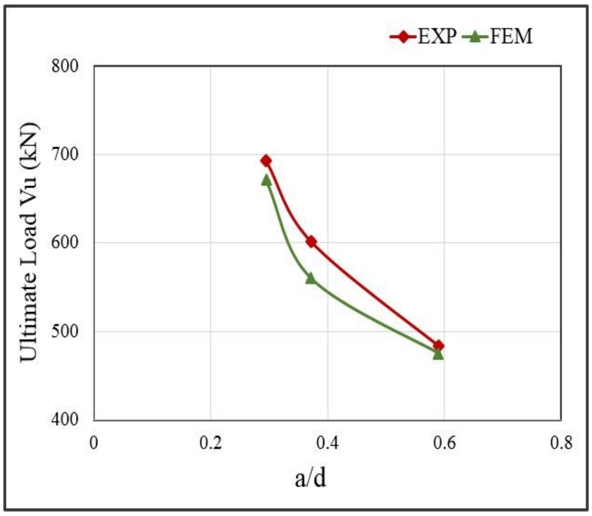
5.1.4 Effect of a/d Ratio on the Ultimate Strength of the Brackets

Series number five, indicates a good agreement with the concept that proved through the experimental study as shown in Fig (8 a), where it was found that decreasing a/d ratio from 0.621 to 0.171 led to an increase in the ultimate strength of 47.08% for the normal concrete brackets.

As for the series six with specimen T3-K3 which contains stirrups, it is found that decreasing the a/d ratio from 0.590 to 0.295 leads to increasing the ultimate strength by 29.26% for the normal concrete brackets. as shown in Fig(8b).



a- Specimens without Stirrups

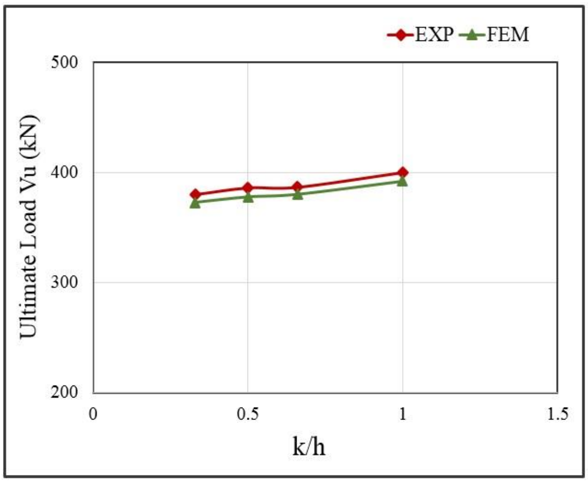


b- Specimens with Stirrups

Fig (8) Variation the Ultimate Load with the Ratio of Shear Span to Effective Depth (a/d)

5.1.5 Effect of (k/h) Ratio on the Ultimate Strength of the Brackets

The results of series seven with the specimen T3-K1 clarified the effect of k/h ratio on the ultimate strength of the brackets, it is found that increasing k/h from 0.33 to 1.00 led to increase the ultimate strength by about 4.94% as shown in Fig (9).



Fig(9) Variation of the Ultimate Load with k/h Ratio

6- Stress in Main Reinforcement

The values of the shear strength at the yield point of main reinforcement and the stress of main reinforcement at failure have been arranged in Table (3), and Fig (10) shows the finite element results for stress in reinforcement. By observing the finite element results and comparison with the experimental results, a good agreement is found between them with small differences although some experimental values of (Vy) were not available. In the first series the difference was in the specimen T1-K3 where in the experimental study this specimen reached the yield point at the failure while in the finite element analysis this specimen failed before the main reinforcement reached the yield point. In the second series, the difference was in the specimen T2-K3 where it yielded before failure in finite element analysis while in the experimental study, this specimen failed before yielding of the main reinforcement, in the series number three the agreement was very good between the experimental and finite element values of shear strength when the main reinforcement reached the yield point and the difference was 7%. In the series four, the experimental value of (Vy) for the specimen T4-K1 was not measured while in the finite element analysis the reinforcement yielded at the ultimate load. In the series five, the difference ratio was only 1%, for the series number six, the difference in the specimen T6-K1 was 11% and specimen T6-K2 was failed before the main reinforcement reached the yield point while in the experimental study, the main reinforcement was yielded at the failure, in the last series number seven, there was good agreement between the results of FE and the experimental although the specimen T7-K1 failed before the main reinforcement yielded while in the experimental the shear strength when the main reinforcement reached the yield point was not measured.

In general, the difference ratios was very small and thus giving indication that the agreement between the finite element and the experimental results was very good.

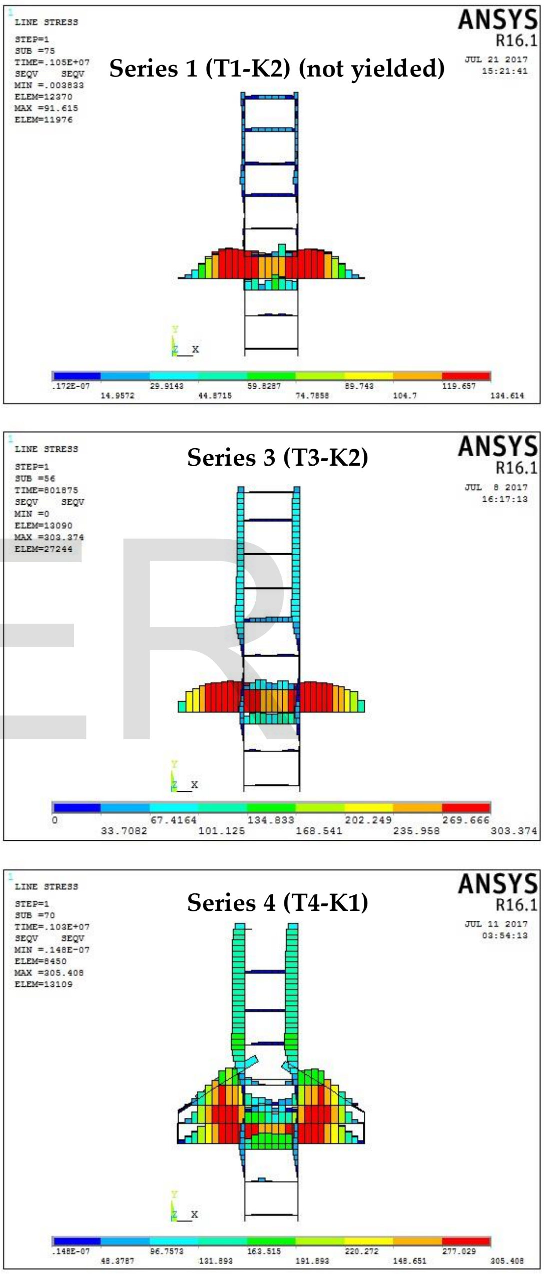


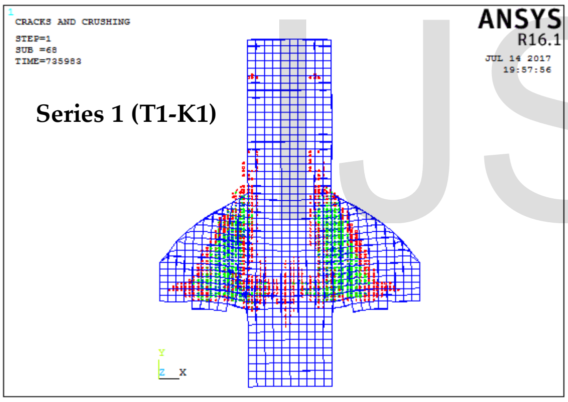
Fig (10) Stress in the Reinforcement for some Specimens at Failure

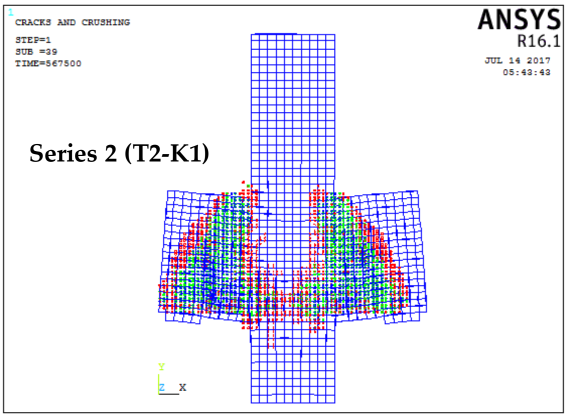
7- Cracks Patterns

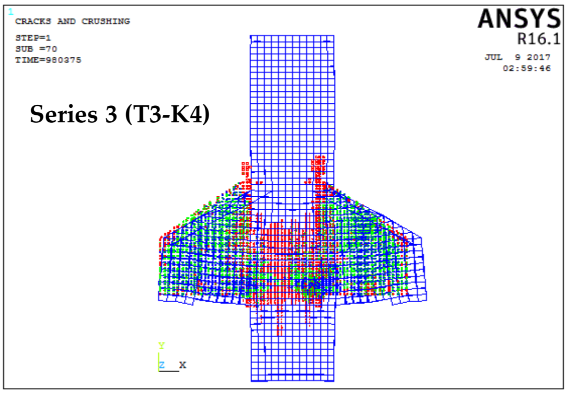
Table (2), contains the failure mode or type for the specimens. The specimens of series one and two have the similar cracks patterns, by observing the crack patterns from ANSYS, first crack were flexural crack. They appeared from the junction of the horizontal face of the bracket and column face while these cracks continued to form or were well to developed, a diagonal crack appeared from the load plate toward the junction of the sloping face of the bracket these cracks led the brackets to fail by Diagonal Tension Failure. This mode of the failure was expected because these specimens have no stirrups.

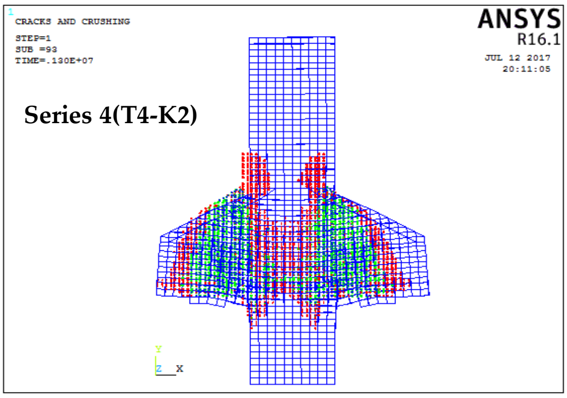
In the specimen T2-K1 the first flexural crack was rapid and more sudden than the other specimens because it has lower main reinforcement ratio.

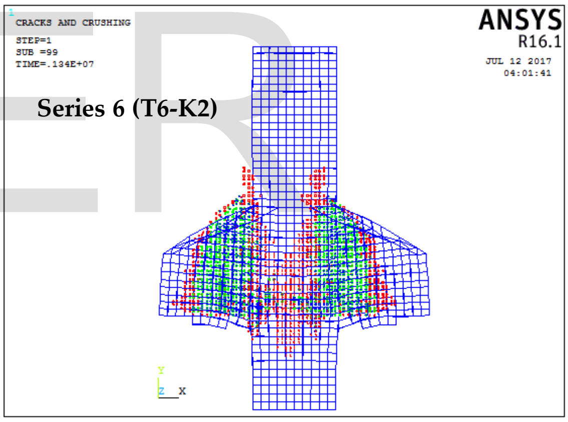
For the specimens of series three, four and six that contained stirrups, all specimens failed by Beam Shear Failure, except the specimen T3-K1 that failed by Diagonal Tension Failure, due to the provided stirrups reinforcement which eliminate the possibility of a premature diagonal tension failure. The first cracks were flexural cracks and with increasing the load these cracks developed and expanded on its base leading to failure by "Beam Shear". The remaining specimens of the series five and seven have similar cracks patterns to series one and two where the failure modes that prevailed were Diagonal Tension Failure. The cracks patterns at failure are illustrated in Fig (11)











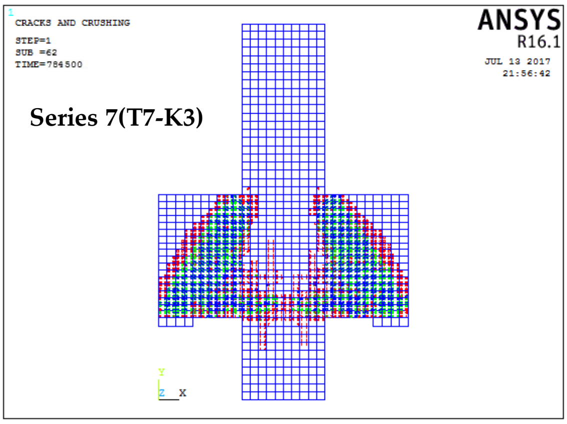


Fig (11) Crack Pattern at Failure

8- Load-Deflection Behavior

The load-deflection behavior is traced for the entire load range, and the corresponding curves are plotted up to failure for the case study under consideration. The deflection values for the finite element models are calculated at the center of each column. The values of the deflection have been arranged in Table (2).

The parameters shown to affect the ultimate load of the bracket specimens, are also considered here for their effect on the load-deflection behavior, it is observed that increasing the compressive strength, ratio of main reinforcement, ratio of stirrups reinforcement and ratio of outside depth to total depth of the brackets (k/h) led to decrease the deflection at the same applied load as shown figures (12), (13), (14) and (15)

Regarding the effect of a/d, it is found that the deflection decreased when decreased the ratio of a/d as shown in fig (16)

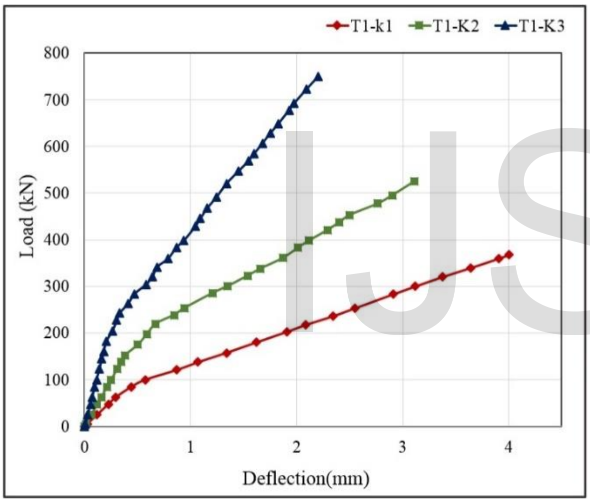


Fig (12) Effect of $f'c$ on Load-Deflection Behavior

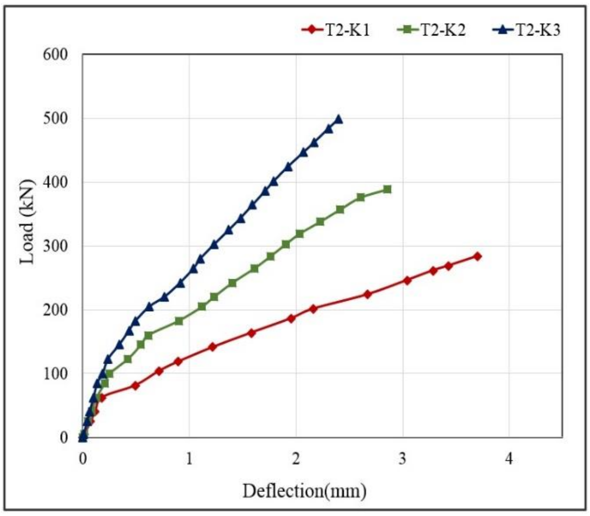
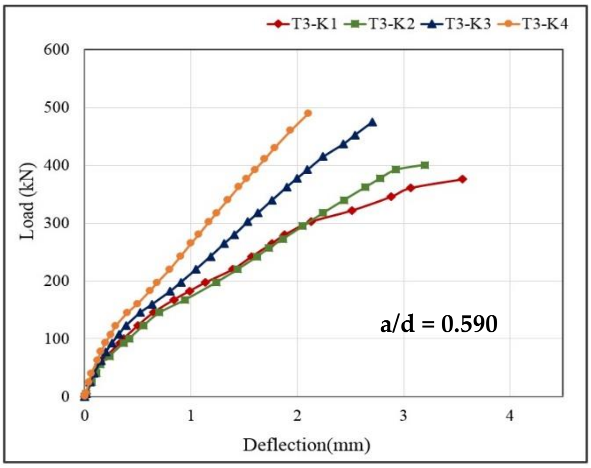


Fig (13) Variation of Load-Deflection Behavior with Main Reinforcement



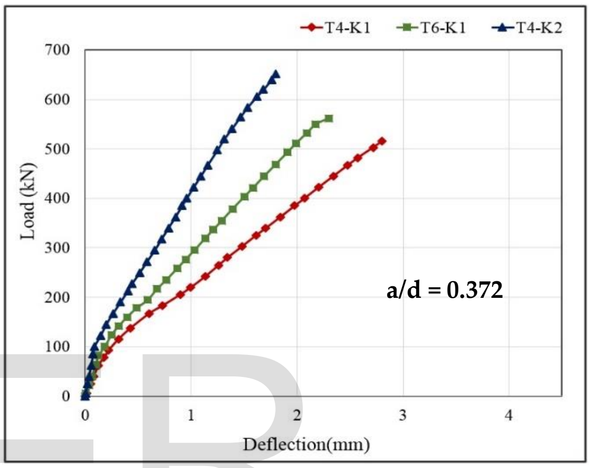


Figure (14) Variation of Load-Deflection Behavior with Stirrups Reinforcement

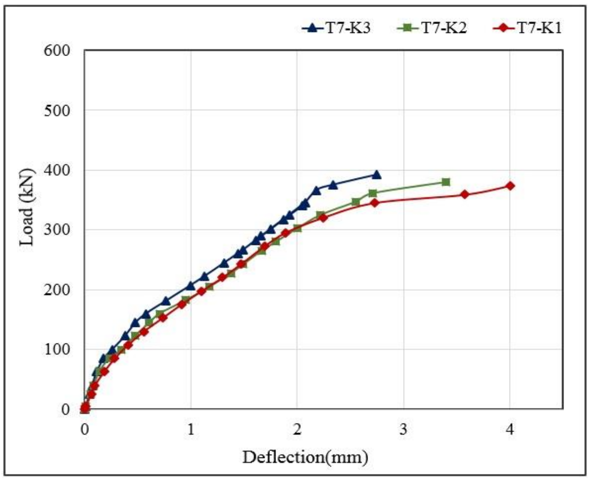


Fig (15) Variation of the Load-Deflection Behavior with k/h ratio

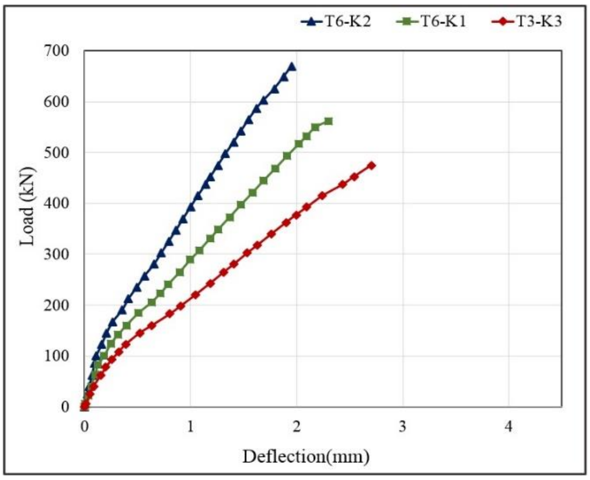


Fig (16) Variation of the Load-Deflection Behavior with a/d ratio

9- Conclusion

Based on the results of **Finite Element Method** that were obtained through computer program (ANSYS 16.1), it is found that the results have good agreement with the experimental results and it is led that the **Finite Element Method** is good method to evaluate the ultimate strength and study the behavior of the reinforced concrete brackets and the maximum difference ratio between the two methods was 7% of the ultimate strength in specimen T6-K1, The following conclusions can be made to evaluate the behavior and the ultimate strength for the 20 specimens that have been analyzed.

1- The increasing of the compressive strength, main and secondary reinforcement and the ratio of depth of brackets to total depth led to increase the ultimate strength of the normal concrete brackets
2- The ratio of shear span to effective depth (a/d) is significant in evaluation of the ultimate strength of the brackets where it is found that decrease of the ratio of a/d led to increase the ultimate load.
3- There was not a clear relationship between the stress in reinforcement and the load at the failure of ultimate strength because some specimens yielded before reaching the ultimate load and the others failed without yield of the reinforcement.
4- The first cracks were flexural cracks formed from the column-bracket intersection for all specimens
5- Providing the stirrups caused to change the failure mode from diagonal tension failure to beam-shear failure for all specimens, so the specimens with stirrups reinforcement failed by beam- shear failure.
6- The equation of ACI318-77 gave good factor of safety according to the comparison with the finite element result where it reached to 1.72, and this equation considered the effect of all variables which were considered in this study except the effect of k/d ratio, where it was found that increasing the compressive strength of concrete, and amount of main and secondary reinforcement are leading to increase the ultimate load, and found that decreasing a/d ratio also led to increases the ultimate load.

10- References

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Table (1) Details of Case Study [6]

Series No.	*Spec No.*	*a/d*	*k/h*	*ρ %*	*ρ_h %*	*f′c MPa*	*Main Reinforcement*		*Stirrups*	
							Amount (mm)	*f_y MPa*	*Amount (mm)*	*f_{yh} MPa*
1	T1-K1	0.297	0.33	1.23	0	14	5-D16	310	Nil	-
	T1-K2	0.297	0.33	1.23	0	28	5-D16	310	Nil	-
	T1-K3	0.297	0.33	1.23	0	44	5-D16	310	Nil	-
2	T2-K1	0.525	1.00	0.48	0	27	2-D16	315	Nil	-
	T2-K2	0.525	1.00	0.93	0	27	4-D16	315	Nil	-
	T2-K3	0.525	1.00	1.23	0	27	5-D16	315	Nil	-
3	T3-K1	0.590	0.5	0.93	0	31	4-D16	375	Nil	-
	T3-K2	0.590	0.5	0.93	0.34	31	4-D16	303	3-D8	344
	T3-K3	0.590	0.5	0.93	0.62	31	4-D16	303	5-D8	318
	T3-K4	0.590	0.5	0.93	0.93	31	4-D16	303	7-D8	323
4	T4-K1	0.372	0.5	0.93	0.34	31	4-D16	305	3-D8	392
	T4-K2	0.372	0.5	0.93	0.93	31	4-D16	305	7-D8	338
5	T5-K1	0.171	0.33	0.93	0	25	4-D16	300	Nil	-
	T5-K2	0.372	0.33	0.93	0	25	4-D16	326	Nil	-
	T5-K3	0.621	0.33	0.93	0	25	4-D16	307	Nil	-
6	T6-K1	0.372	0.5	0.93	0.62	31	4-D16	305	5-D8	392
	T6-K2	0.295	0.5	0.93	0.62	31	4-D16	327	5-D8	338
7	T7-K1	0.590	0.33	0.93	0	31	4-D16	365	Nil	-
	T7-K2	0.590	0.66	0.93	0	31	4-D16	305	Nil	-
	T7-K3	0.590	0.1	0.93	0	31	4-D16	306	Nil	-

Table (2) Theoretical and Experimental Results

Series No.	*Spec No.*	*Ultimate Load Vu (kN)*			*1st Crack Load FEM (kN)*	**Deflection at Ultimate Load** *Δ(FEM) (mm)*	$\frac{Vu(FEM)}{Vu(EXP)}$	*Type of Failure*
		FEM	*EXP*	*ACI*				
1	T1-K1	367.5	375.2	247.74	85	4	0.98	D.T
	T1-K2	525	547.13	350.36	175	3.1	0.96	D.T
	T1-K3	750	762.95	439.2	220	2.2	0.98	D.T
2	T2-K1	283.87	286.81	219.11	70	3.7	0.98	D.T
	T2-K2	388.65	400	267.38	85	2.85	0.97	D.T
	T2-K3	499.37	505.58	299.56	100	2.4	0.98	D.T
3	T3-K1	377.62	386.15	272.2	62.5	3.55	0.95	D.T
	T3-K2	400.93	420.5	320.24	70	3.2	0.97	B.S
	T3-K3	475	483.55	347.54	77.5	2.7	0.98	B.S
	T3-K4	490	486.41	367.2	92.5	2.1	1.007	B.S
4	T4-K1	517.5	533.96	369.75	77.5	2.8	0.96	B.S
	T4-K2	650	664.6	423.97	107.5	1.8	0.97	B.S
5	T5-K1	444.12	445.13	317.07	107.5	2.3	0.99	D.T
	T5-K2	384.46	395.89	282.22	70	2.8	0.97	D.T
	T5-K3	235	250	230.06	55	3.7	0.94	D.T
6	T6-K1	560	601.57	400	94	2.3	0.93	B.S
	T6-K2	671.5	693.24	418.93	122.5	1.95	0.96	B.S
7	T7-K1	372.6	380.41	272.2	62.5	4	0.97	D.T
	T7-K2	380	386.72	272.2	77.5	3.44	0.98	D.T
	T7-K3	392	400	272.2	92.5	2.75	0.98	D.T

D.T – Diagonal Tension Failure B.S – Beam Shear Failure

FEM = Finite Element Method EXP = experimental Values

Table (3) Theoretical and Experimental Values of Vy

Series No.	*Spec No.*	*Shear Strength at the Yield of Main Steel Vy (kN)*		$\frac{Vy(FEM)}{Vy(EXP)}$	**Stress of Main Reinforcement at Failure (MPa)**
		FEM	*EXP*		
1	T1-K1	[1]	[1]	-	94.430
	T1-K2	[1]	[1]	-	143.614
	T1-K3	[1]	762.95	-	169.849
2	T2-K1	220	200	1.1	315.414
	T2-K2	355.76	349.45	1.018	325.276
	T2-K3	461.5	[1]	-	322.322
3	T3-K1	[1]	[2]	-	253.283
	T3-K2	390.87	420.5	0.93	303.347
	T3-K3	463.44	483.55	0.95	303.489
	T3-K4	479.8	486.41	0.98	305.848
4	T4-K1	517.5	[2]	-	305.408
	T4-K2	650	664.6	0.97	305.088
5	T5-K1	444.12	445.133	0.99	415.393
	T5-K2	[1]	[1]	-	212.099
	T5-K3	[1]	[1]	-	145.184
6	T6-K1	540	601.57	0.89	305.888
	T6-K2	[1]	693.24	-	281.955
7	T7-K1	[1]	[2]	-	310.179
	T7-K2	380	378.13	1.005	305.109
	T7-K3	343.75	347.18	0.99	306.169

[1] Did not yield

[2] The value not measured in the experimental study